## Collective Noise Contrastive Estimation for Policy Transfer Learning

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## Xbox Radio Track Playing Problem

For each session:

- Seed: the user sets a seed artist
- Action: the policy plays tracks one by one to the user
- Feedback: if the user does not like the current track, he can push the 'skip' button


## Problem:

- Design a policy to maximise user satisfaction, quantified as policy reward score


## Two data sources:

- Auxiliary data: user self-generated playlists (can be regarded as positive examples)
- Target data: user feedback (skip or listen) given the historic radio playing


## Model

## Notations:

- Context $i$, selected artist $j$, reward $r$
- Score function $s_{\theta}(i, j, r)=r \cdot w_{i}^{T} w_{j}+b_{j}$

Softmax-based Stochastic Policy:

- Conditional probability
$P_{\theta}(j, r \mid i)=e^{s_{\theta}(i, j, r)} / \sum_{r^{\prime}} \sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, r^{\prime}\right)}$

- Radio selection
$P_{\theta}(j \mid i, r=1)=e^{s_{\theta}(i, j, 1)} / \sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, 1\right)}$


## Objective 1: Maximising data generation likelihood

- Playlist data likelihood: $\quad \mathcal{L}_{P}\left(P_{\theta}\right)=\prod_{(i, j, 1) \in D_{P}} P_{\theta}(j \mid i, r=1)$
- Radio data likelihood: $\quad \mathcal{L}_{R}\left(P_{\theta}\right)=\prod_{(i, j, r) \in D_{R}} P_{\theta}(j, r \mid i)$
- Joint optimisation:

$$
\begin{aligned}
& \max _{\theta} \frac{\alpha}{\left|D_{R}\right|} \log \mathcal{L}_{R}\left(P_{\theta}\right)+\frac{1-\alpha}{\left|D_{P}\right|} \log \mathcal{L}_{P}\left(P_{\theta}\right) \\
= & \max _{\theta} \frac{\alpha}{\left|D_{R}\right|} \sum_{(i, j, r) \in D_{R}} \log \frac{e^{s_{\theta}(i, j, r)}}{\sum_{r^{\prime}} \sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, r^{\prime}\right)}}+\frac{1-\alpha}{\left|D_{P}\right|} \sum_{(i, j, 1) \in D_{P}} \log \frac{e^{s_{\theta}(i, j, 1)}}{\sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, 1\right)}}
\end{aligned}
$$

Objective 2: Maximising inverse propensity score (IPS) based policy value

- Expected reward of a policy: $\mathbb{E}_{i}\left[\mathbb{E}_{P_{\theta}\left(j \mid i, r^{\prime}=1\right)}\left[\overrightarrow{r_{i}}[j]\right]\right]$
- IPS policy value on radio data:

$$
\begin{aligned}
\hat{V}_{\mathrm{ips}}\left(P_{\theta}\right) & =\frac{1}{\left|D_{R}\right|} \sum_{(i, j, r) \in D_{R}} \frac{r P_{\theta}\left(j \mid i, r^{\prime}=1\right)}{P_{D}(j \mid i)}=\frac{1}{\left|D_{R}\right|} \sum_{(i, j, r) \in D_{R}} \frac{r}{P_{D}(j \mid i)} \frac{e^{s_{\theta}(i, j, 1)}}{\sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, 1\right)}} \\
& >\frac{1}{\left|D_{R}\right|} \sum_{(i, j, r) \in D_{R}} \frac{r}{P_{D}(j \mid i)} \log \frac{e^{s_{\theta}(i, j, 1)}}{\sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, 1\right)}}=\tilde{V}_{\mathrm{ips}}\left(P_{\theta}\right) \quad \text { [lower bound] }
\end{aligned}
$$

- Joint optimisation:

$$
\begin{aligned}
& \max _{\theta} \alpha \tilde{V}_{\mathrm{ips}}\left(P_{\theta}\right)+\frac{1-\alpha}{\left|D_{P}\right|} \log \mathcal{L}_{P}\left(P_{\theta}\right) \\
= & \max _{\theta} \frac{\alpha}{\left|D_{R}\right|} \sum_{(i, j, r) \in D_{R}} \frac{r}{P_{D}(j \mid i)} \log \frac{e^{s_{\theta}(i, j, 1)}}{\sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, 1\right)}}+\frac{1-\alpha}{\left|D_{P}\right|} \sum_{(i, j, 1) \in D_{P}} \log \frac{e^{s_{\theta}(i, j, 1)}}{\sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, 1\right)}}
\end{aligned}
$$



Gradient calculation via noise contrastive estimation (NCE)

- Expensive gradient calculation on softmax function:

$$
\frac{\partial}{\partial \boldsymbol{\theta}} \log \frac{e^{s_{\theta}(i, j, r)}}{\sum_{j^{\prime}} e^{s_{\theta}\left(i, j^{\prime}, r\right)}}=\frac{\partial s_{\boldsymbol{\theta}}(i, j, r)}{\partial \boldsymbol{\theta}}-\mathbb{E}_{P_{\boldsymbol{\theta}}\left(j^{\prime} \mid i, r\right)}\left[\frac{\partial s_{\boldsymbol{\theta}}\left(i, j^{\prime}, r\right)}{\partial \boldsymbol{\theta}}\right]
$$

- NCE idea: define a loss function to quantify how likely the policy will separate a data point from $k$ noise data points generated from a known noise probabilistic distribution.

$$
\begin{aligned}
\mathcal{L}_{\mathrm{NCE}}^{(i, j, r)}(\boldsymbol{\theta}) & =\log \frac{P_{\theta}(j \mid i, r)}{P_{\theta}(j \mid i, r)+k P_{n}(j)}+\sum_{m=1}^{k} \log \frac{k P_{n}\left(j_{m}\right)}{P_{\theta}\left(j_{m} \mid i, r\right)+k P_{n}\left(j_{m}\right)} \\
\left.\frac{\partial}{\partial \boldsymbol{\theta}} \mathcal{L}_{\mathrm{NCE}}^{(i, j, r)} \boldsymbol{\theta}\right) & =\frac{k P_{n}(j)}{e^{s_{\boldsymbol{\theta}}(i, j, r)}+k P_{n}(j)} \frac{\partial s_{\boldsymbol{\theta}}(i, j, r)}{\partial \boldsymbol{\theta}} \sum_{m=1}^{k} \frac{e^{s_{\boldsymbol{\theta}}\left(i, j_{m}, r\right)}}{e^{s_{\boldsymbol{\theta}}^{\left(i, j_{m}, r\right)}+k P_{n}\left(j_{m}\right)} \frac{\partial s_{\boldsymbol{\theta}}\left(i, j_{m}, r\right)}{\partial \boldsymbol{\theta}}}
\end{aligned}
$$

- when $k \rightarrow+\infty$, the gradient $\frac{\partial}{\partial \boldsymbol{\theta}} \mathcal{L}_{\mathrm{NCE}}^{(i, j, r)}(\boldsymbol{\theta}) \rightarrow \frac{\partial}{\partial \boldsymbol{\theta}} \log \frac{e^{s_{\theta}(i, j, r)}}{\sum_{j^{\prime}} e^{s} e^{\left(i, j^{\prime}, r\right)}}$.


## Experiment on Xbox Music Playlist \& Radio Data

Datasets:

- Playlist: 20.3k playlists with 722.7 k transitions on 1.81 k artists
- Radio: 97.6k transation sequences on 1.44 k artists (1.03k artists occur in playlists) Performance with Objective 1:

| Algorithm | log-likelihood |
| :---: | :---: |
| Random | -7.7932 |
| Popularity | -5.8009 |
| NCE-Playlist | -10.3978 |
| NCE-Radio | -5.5197 |
| NCE-Collective | $\mathbf{- 5 . 5 0 7 2}$ |



Performance with Objective 2:

| Algorithm | IPS Value |
| :---: | :---: |
| Random | 0.0687 |
| Popularity | 0.0747 |
| SameArtist | -0.3088 |
| NCE-Playlist | 0.0695 |
| NCE-Radio | 0.3912 |
| NCE-Collective | $\mathbf{0 . 4 1 1 1}$ |




## Case studies:



